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Dispersive Treatment of Weak Decays and Final-State Interactions in Model Theories*

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A model, in which a heavy fermion B is added to the Lee model and weakly coupled to V and θ is considered. Decay amplitudes for $B\to V+\theta$ and $B\to N+\theta+\theta$ are evaluated by dispersion theoretic methods. The absorptive part of these amplitudes incorporate contributions from one- and two-boson intermediate states. Attention is focused on the question of how well founded is the usual approximate treatment of absorptive amplitudes, which neglects the higher mass states (here the two-boson states) with respect to the lowest mass (here the one-boson) states. It is shown that in a dispersion scheme which involves sufficient subtraction to give completely finite results, and which involves as many arbitrary constants as the theory allows, the one-particle contributions to the absorptive parts of the $B\to N+\theta+\theta'$ amplitude diverge logarithmically and that these logarithmic divergences are cancelled by the two-particle contributions to this amplitude.

I. INTRODUCTION

In a recent paper, one of us[1] discussed the decay of a heavy fermion, called a B particle, which had been added to the Lee model and which was weakly coupled to V and θ by the Hamiltonian

$$H_w=G_0\sum_{\mathbf{k}}\frac{u(k)}{(2\omega_k)^{1/2}}(b^\dagger va_{\mathbf{k}}+v^\dagger ba_{\mathbf{k}}^\dagger). \quad (1)$$

The calculation was carried out in a dressed particle picture and the amplitudes for $B\to V+\theta$ and $B\to N+\theta+\theta$ decay were represented as spectral sums involving contributions from the one-particle $(V-\theta)$ and the two-particle $(N-\theta-\theta)$ states. Although, as expected, the perturbative series (in g_ρ, the renormalized strong coupling constant) consisted of convergent integrals only, the one- and the two-particle parts separately contributed divergent terms which, however, combined to give finite and correct results.

This situation, in the event that it also arises in the dispersion theoretic treatment of this problem has obvious and serious implications for the approximate treatment of the absorptive part of amplitudes in which the "lowest mass" contributions are retained, whereas the "higher mass" terms are dropped on the assumption that they will be dominated by the former. The present calculation was undertaken to inquire whether such cancellations do indeed occur in a dispersion theoretic evaluation of the amplitudes for B decay.

II. $B\to V+\theta$ DECAY

To lowest order in the weak coupling constant $D_v(\omega)$, the amplitude for $B\to V+\theta$ decay, is given by $\langle V\theta_{\mathrm{p}}^{(\mathrm{out})}|H_w|B\rangle$. Contraction on θ_{p} gives

$$D_v(\omega)=\langle V|a_{\mathrm{p}}H_w|B\rangle+\int_{-\infty}^{+\infty}dt\,\theta(t)\langle V|A_{\mathrm{p}}H_w|B\rangle. \quad (2)$$

It is convenient to write

$$D_v(\omega_p)=u(p)(2\omega_p)^{-1/2}F(\omega_p),$$

where

$$F(\omega)=G_0\langle V|v^\dagger|0\rangle+\phi(\omega), \quad (3a)$$

$$\phi(\omega)=i\int_{-\infty}^{+\infty}dt\,\theta(t)e^{i\omega t}\langle V|\,j(t)H_w|B\rangle, \quad (3b)$$

and $j(t)$ is specified by

$$\dot{A}_{\mathbf{k}}(t)=ie^{i\omega_k t}u(k)(2\omega_k)^{-1/2}j(t)$$

and by

$$\langle N|\,j(0)\,|V\rangle=g_\rho.$$

The absorptive part of $\phi(\omega)$ is given by

$$\phi_a(\omega)=\pi\sum\nolimits_l\delta(E_l-m-\omega)\langle V|\,j(0)\,|l\rangle\langle l|H_w|B\rangle, \quad (4)$$

where $\sum_l$ indicates summation over a complete set of states. Since the calculation is to first order in the weak-coupling constant, the complete spectrum for the strong-coupling problem is the proper one to use. Due to the selection rules that are operative in the Lee model, only the sector containing the $V\theta$ and the $N\theta\theta$ states (we here use the outgoing states) contributes to the sum. $\phi(\omega)$ can be written as the dispersion integral[2]

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[1] K. Haller, J. Math. Phys. 4, 323 (1963).

[2] J. D. Jackson, in *Dispersion Relations*, edited by G. R. Screaton (Interscience Publishers, Inc., New York, 1960). See also M. L. Goldberger, in *Dispersion Relations and Elementary Particles*, edited by C. DeWitt and R. Omnes (John Wiley & Sons, Inc., New York, 1960).

$\phi(\omega)=(\pi)^{-1}\int\phi_a(\omega')[\omega'-\omega-i\eta]^{-1}d\omega'$; $\phi(\omega)$ becomes

$$\phi(\omega_p)=\sum_{\mathbf{k}}\frac{\langle V|j(0)|V\theta_{\mathbf{k}}^{(\text{out})}\rangle D_v(\omega_k)}{\omega_k-\omega_p-i\eta}+\sum_{\mathbf{k},\mathbf{k}'}\frac{\langle V|j(0)|N\theta_{\mathbf{k}}\theta_{\mathbf{k}'}^{(\text{out})}\rangle D_n(\omega_k,\omega_{k'})}{\omega_k+\omega_{k'}-\omega_p-i\eta}, \quad (4)$$

where $D_n(\omega_k,\omega_{k'})$ is the amplitude for $B\to N+\theta_{\mathbf{k}}+\theta_{\mathbf{k}'}$ decay.

To represent the amplitude $D_v(\omega)$ in terms of a renormalized weak-coupling constant it is necessary to make a subtraction; this is because the renormalization of the weak-coupling constant by the strong interaction involves the renormalization of decay vertex graphs which never arise in the Lee model itself, so that the ratio of the renormalized to the unrenormalized weak-coupling constant, (G_ρ/G_0) cannot be expressed in terms of the renormalization constants of the Lee model alone. We write

$$F(\omega_p)-F(0)=\omega_p\sum_{k}\frac{\langle V|j(0)|V\theta_{\mathbf{k}}^{(\text{out})}\rangle D_v(\omega_k)}{\omega_k(\omega_k-\omega_p-i\eta)}+\omega_p\sum_{k,k'}\frac{\langle V|j(0)|N\theta_{\mathbf{k}}\theta_{\mathbf{k}'}^{(\text{out})}\rangle D_n(\omega_k,\omega_{k'})}{(\omega_k+\omega_{k'})(\omega_k+\omega_{k'}-\omega_p-i\eta)}, \quad (5)$$

and set $F(0)=G_\rho$. The matrix elements appearing in Eq. (5) are easily expressed in terms of $T_v(\omega)$ and $R(\omega)$, the transition matrix elements for the elastic process $V+\theta_\omega\to V+\theta_\omega$ and for the inelastic process $V+\theta_\omega\to N+\theta_1+\theta_2$, respectively, both evaluated on the energy shell. We finally write the integral equation

$$F(\omega_p)=G_\rho-\omega_p\sum_{\mathbf{k}}\frac{(2\omega_k)^{1/2}}{u(k)}\frac{T_v^*(\omega_k)D_v(\omega_k)}{\omega_k(\omega_k-\omega_p-i\eta)}-\omega_p\sum_{\mathbf{k},\mathbf{k}'}\frac{[2(\omega_k+\omega_{k'})]^{1/2}}{u(k,k')}\times\frac{R^*(\omega_k+\omega_{k'})D_n(\omega_k,\omega_{k'})}{(\omega_k+\omega_{k'})(\omega_k+\omega_{k'}-\omega_p-i\eta)}. \quad (6)$$

III. $B\to N+\theta+\theta$ DECAY

The decay amplitude $D_n(\omega_q,\omega_p)$, to lowest order in G_ρ is given by

$$D_n(\omega_q,\omega_p)=\langle N\theta_{\mathbf{q}}\theta_{\mathbf{p}}^{(\text{out})}|H_w|B\rangle. \quad (7)$$

After contraction on $\theta_{\mathbf{p}}$ this becomes

$$D_n(\omega_q,\omega_p)=(2)^{-1/2}\left[\int_{-\infty}^{+\infty}dt\,\theta(t)\langle N\theta_{\mathbf{q}}^{(\text{out})}|A_pH_w|B\rangle+\langle N\theta_{\mathbf{q}}^{(\text{out})}|a_pH_w|B\rangle\right]. \quad (7\text{a})$$

We will refer to the two terms on the right-hand side (r.h.s.) of Eq. (7a) as $\alpha(\omega_q;\omega_p)$ and $\beta(\omega_q;\omega_p)$, respectively. $\alpha(\omega_q;\omega_p)$ can be written as $-u(p)[2\sqrt{\omega_p}]^{-1}\times\chi(\omega_q;\omega_p)$ and $\chi(\omega_q;\omega_p)$ then becomes

$$\chi(\omega_q;\omega_p)=i\int_{-\infty}^{+\infty}dt\,\theta(t)e^{i\omega_pt}\langle N\theta_q^{(\text{out})}|j(t)H_w|B\rangle. \quad (8)$$

The absorptive part, $\chi_a(\omega_q;\omega_p)$ is written

$$\chi_a(\omega_q;\omega_p)=\pi\sum_l\delta(\omega_p+m+\omega_q-E_l)\times\langle NA_q^{(\text{out})}|j(0)|l\rangle\langle l|H_w|B\rangle,$$

where the summation again extends over the $V-\theta$, $N\theta\theta$ sector of the Lee model. The dispersion relation leads to

$$\alpha(\omega_q;\omega_p)=\frac{u(p)}{2(\omega_p)^{1/2}}\left\{\sum_{\mathbf{k}}\frac{\mathcal{R}(\mathbf{q};\mathbf{k})D_v(\omega_k)}{\omega_k-\omega_q-\omega_p-i\eta}+\sum_{\mathbf{k},\mathbf{k}'}\frac{\mathcal{S}(\mathbf{q};\mathbf{k},\mathbf{k}')D_n(\omega_k,\omega_{k'})}{\omega_k+\omega_{k'}-\omega_q-\omega_p-i\eta}\right\}, \quad (9)$$

where $\mathcal{R}(\mathbf{q};\mathbf{k})$ and $\mathcal{S}(\mathbf{q};\mathbf{k},\mathbf{k}')$ denote $\langle N\theta_{\mathbf{q}}^{(\text{out})}|j(0)|\times V\theta_{\mathbf{k}}^{(\text{out})}\rangle$ and $\langle N\theta_{\mathbf{q}}^{(\text{out})}|j(0)|N\theta_{\mathbf{k}}\theta_{\mathbf{k}'}^{(\text{out})}\rangle$, respectively. Integral equations for $\mathcal{R}(\mathbf{q};\mathbf{k})$ and $\mathcal{S}(\mathbf{q};\mathbf{k},\mathbf{k}')$ can be written by systematically commuting the outgoing boson annihilation operators from the left of $j(0)$ to its right, and writing dispersion relations for the resulting commutators which later disappear at $t=0$.[3] These integral equations are

$$\mathcal{R}(\mathbf{q};\mathbf{k})=g_\rho\delta_{\mathbf{q},\mathbf{k}}+\frac{u(q)}{(2\omega_q)^{1/2}}\left\{g_\rho\frac{(2\omega_k)^{1/2}}{u(k)}T_v^*(\omega_k)\left[\frac{1}{\omega_q}+\frac{1}{\omega_k-\omega_q-i\eta}\right]+\sum_{\kappa}\frac{(2\omega_\kappa)^{1/2}}{u(\kappa)}T_n^*(\omega_\kappa)\mathcal{R}(\boldsymbol{\kappa};\mathbf{k})\left[\frac{1}{\omega_k-\omega_\kappa-\omega_q-i\eta}-\frac{1}{\omega_\kappa-\omega_q-i\eta}\right]\right\} \quad (10)$$

[3] The quantity $R(q;k)$ is almost identical to $F(\omega,\omega')$ in R. Amado, Phys. Rev. **122**, 696 (1961); $F(\omega,\omega')$ differs in being defined for "in" instead of "out" states, and by trivial multiplicative constants.

and

$$\mathcal{S}(\mathbf{q};\mathbf{k},\mathbf{k}')=-(2)^{-1/2}\left[\delta_{\mathbf{q},\mathbf{k}'}\frac{(2\omega_k)^{1/2}}{u(k)}T_n^*(\omega_k)+\delta_{\mathbf{q},\mathbf{k}}\frac{(2\omega_{k'})^{1/2}}{u(k')}T_n^*(\omega_{k'})\right]$$
$$+\frac{u(q)}{(2\omega_q)^{1/2}}\left\{g_\rho\frac{[2(\omega_k+\omega_{k'})]^{1/2}}{u(k,k')}\mathcal{R}^*(\omega_k+\omega_{k'})\left[\frac{1}{\omega_q}+\frac{1}{\omega_k+\omega_{k'}-\omega_q-i\eta}\right]\right.$$
$$\left.+\sum_\kappa\frac{(2\omega_\kappa)^{1/2}}{u(\kappa)}T_n^*(\omega_\kappa)\mathcal{S}(\boldsymbol{\kappa};\mathbf{k},\mathbf{k}')\left[\frac{1}{\omega_k+\omega_{k'}-\omega_\kappa-\omega_q-i\eta}-\frac{1}{\omega_\kappa-\omega_q-i\eta}\right]\right\}.\quad(10a)$$

$\beta(\omega_q;\omega_p)$ is evaluated by contracting on $\theta_\mathbf{q}$. This leads to

$$\beta(\omega_q;\omega_p)=(2)^{-1/2}[\langle N|a_\mathbf{q}a_\mathbf{p}H_w|B\rangle+\langle N|\dot{A}_\mathbf{q}a_\mathbf{p}H_w|B\rangle].\quad(11)$$

The first term on the r.h.s. of Eq. (11) vanishes, since H_w is linear in boson operators and since $a_\mathbf{k}|B\rangle=0$.[4] The elimination of this term is essential. Dispersion schemes which circumvent this step and culminate in equations which still contain $\langle N|a_\mathbf{q}a_\mathbf{p}H_w|B\rangle$ implicitly lead to trivial, useless identities instead of soluble integral equations.

It is convenient to write $\beta(\omega_q;\omega_p)=u(q)[2\sqrt{\omega_q}]^{-1}\times\psi(\omega_q;\omega_p)$ in which case $\psi(\omega_q;\omega_p)$ becomes

$$\psi(\omega_q;\omega_p)=i\int_{-\infty}^{+\infty}dt\,\theta(t)e^{i\omega_q t}\langle N|j(t)|l\rangle\langle l|a_\mathbf{p}H_w|B\rangle.\quad(12)$$

The absorptive part, $\psi_a(\omega_q;\omega_p)$, is given by

$$\psi_a(\omega_q;\omega_p)=\pi\sum_l\delta(\omega_q+m-E_l)\times\langle N|j(0)|l\rangle\langle l|a_\mathbf{p}H_w|B\rangle,\quad(12a)$$

where the summation in this case extends over the V, $N-\theta$ sector. It might seem, superficially, that the replacement of $a_\mathbf{p}H_w$ by the commutator $[a_\mathbf{p},H_w]$ in the matrix element $\langle l|a_\mathbf{p}H_w|B\rangle$ would effect a substantial simplification, especially since in this calculation the dressed and bare B operators can be used interchangeably. However such a move must be avoided since it would introduce the bare weak coupling constant into the calculation, thereby obstructing further progress. Instead, $\langle l|a_\mathbf{p}H_w|B\rangle$ is written as the sum

$$\sum_{l'}\langle l|a_\mathbf{p}|l'\rangle\langle l'|H_w|B\rangle.$$

The resulting expression for $\beta(\omega_q;\omega_p)$ is

$$\beta(\omega_q;\omega_p)=\frac{-u(q)}{2\sqrt{\omega_q}}\left\{\sum_\mathbf{k}\left[\frac{g_\rho}{\omega_q}\langle V|a_\mathbf{p}|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle+\sum_\kappa\frac{(2\omega_\kappa)^{1/2}}{u(\kappa)}\frac{T_n^*(\omega_\kappa)\langle N\theta_\kappa^{(\mathrm{out})}|a_\mathbf{p}|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle}{\omega_\kappa-\omega_q-i\eta}\right]D_v(\omega_k)\right.$$
$$\left.+\sum_{\mathbf{k},\mathbf{k}'}\left[\frac{g_\rho}{\omega_q}\langle V|a_\mathbf{p}|N\theta_\mathbf{k}\theta_{\mathbf{k}'}^{(\mathrm{out})}\rangle+\sum_\kappa\frac{(2\omega_\kappa)^{1/2}}{u(\kappa)}\frac{T_n^*(\omega_\kappa)\langle N\theta_\kappa^{(\mathrm{out})}|a_\mathbf{p}|N\theta_\mathbf{k}\theta_{\mathbf{k}'}^{(\mathrm{out})}\rangle}{\omega_\kappa-\omega_q-i\eta}\right]D_n(\omega_k,\omega_{k'})\right\}.\quad(13)$$

In Eq. (13) matrix elements of $a_\mathbf{p}$, taken between various outgoing states, arise and must be evaluated. Let us, for example, consider $\langle V|a_\mathbf{p}|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle$. We know that

$$\langle V\theta_\mathbf{p}^{(\mathrm{out})}|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle=\delta_{\mathbf{p},\mathbf{k}}=\langle V|a_\mathbf{p}|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle+u(p)(2\omega_p)^{-1/2}\xi(\omega_k;\omega_p),\quad(14)$$

where

$$\xi(\omega_k;\omega_p)=i\int_{-\infty}^{+\infty}dt\,\theta(t)e^{i\omega_p t}\langle V|j(t)|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle.$$

Writing a dispersion integral for $\xi(\omega_k;\omega_p)$ leads to

$$\langle V|a_\mathbf{p}|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle=\delta_{\mathbf{p},\mathbf{k}}+\frac{u(p)}{(2\omega_p)^{1/2}}\frac{(2\omega_k)^{1/2}}{u(k)}\frac{T_v^*(\omega_k)}{\omega_k-\omega_p-i\eta}.\quad(15)$$

Similarly, we can show that

$$\langle V|a_\mathbf{p}|N\theta_\mathbf{k}\theta_{\mathbf{k}'}^{(\mathrm{out})}\rangle=\frac{u(p)}{(2\omega_p)^{1/2}}\frac{[2(\omega_k+\omega_{k'})]^{1/2}}{u(k,k')}\frac{\mathcal{R}^*(\omega_k+\omega_{k'})}{\omega_k+\omega_{k'}-\omega_p-i\eta},\quad(15a)$$

$$\langle N\theta_\kappa^{(\mathrm{out})}|a_\mathbf{p}|V\theta_\mathbf{k}^{(\mathrm{out})}\rangle=\frac{-u(p)}{(2\omega_p)^{1/2}}\frac{\mathcal{R}(\boldsymbol{\kappa};\mathbf{k})}{\omega_k-\omega_\kappa-\omega_p-i\eta},\quad(15b)$$

and

$$\langle N\theta_\kappa^{(\mathrm{out})}|a_\mathbf{p}|N\theta_\mathbf{k}\theta_{\mathbf{k}'}^{(\mathrm{out})}\rangle=(2)^{-1/2}[\delta_{\kappa,\mathbf{k}}\delta_{\mathbf{p},\mathbf{k}'}+\delta_{\kappa,\mathbf{k}'}\delta_{\mathbf{p},\mathbf{k}}]-\frac{u(p)}{(2\omega_p)^{1/2}}\frac{\mathcal{S}(\boldsymbol{\kappa};\mathbf{k},\mathbf{k}')}{\omega_k+\omega_{k'}-\omega_\kappa-\omega_p-i\eta}.\quad(15c)$$

[4] The "exact" states are not stationary with respect to the weak interaction. $|B\rangle$, therefore, contains no virtual θ particles.

IV. ITERATION OF DECAY AMPLITUDES

The preceding sections have led to an inhomogeneous system of two coupled linear singular integral equations for the decay amplitudes $D_v(\omega_k)$ and $D_n(\omega_k,\omega_{k'})$. The kernels for these equations are either explicitly known, or else are functions for which other soluble integral equations have been derived. The integral equations for the decay amplitudes, as well as for the auxiliary variables, all have at least an iterative solution which can be explitly generated. In some cases an exact solution can explicitly be given, either in terms of scattering amplitudes alone, or in terms of scattering amplitudes together with other decay amplitudes. For the question of primary import, however, it suffices to generate an iterative solution for $D_v(\omega_k)$ and $D_n(\omega_k,\omega_{k'})$ to the first few orders of g_ρ. Before proceeding with the iteration we note that due to the asymmetric treatment accorded to the $A_{\mathbf{q}}^{(\mathrm{out})}$ and $A_{\mathbf{p}}^{(\mathrm{out})}$ operator in $D_n(\omega_q;\omega_p)$, the expression for the latter amplitude lacks manifest symmetry in ω_q and ω_p; we, therefore, symmetrize it in ω_q and ω_p.

The lowest (zeroth) order of $D_v(\omega_p)$ is the inhomogeneity in Eq. (6) and is given by $D_v^{(0)}(\omega_p)=u(p)(2\omega_p)^{-1/2}G_\rho$. Iterating this equation to the next order gives

$$D_v^{(2)}(\omega_p)=G_\rho\frac{u(p)}{(4\pi^2)}g_\rho^2\left(\frac{\omega_p}{2}\right)^{1/2}\int\frac{k^2dku^2(k)}{\omega_k^3(\omega_k-\omega_p-i\eta)}. \tag{16}$$

Further iteration of Eq. (6) requires the amplitude $D_n(\omega_k,\omega_{k'})$ to first order in g_ρ [$D_v(\omega)$ is an even function of g_ρ, $D_n(\omega,\omega')$ an odd one], which is obtained by iterating Eq. (7a); [cf. also Eqs. (9) and (13)]. The quantities which play the roles of inhomogeneous part in this iteration are integrals involving $D_v^{(0)}(\omega)$. There are two terms which contribute to $D_n(\omega_k,\omega_{k'})$; one originates from Eq. (9) and is contributed by the part of $\mathcal{R}$ which is linear in g_ρ and is the inhomogeneous part of Eq. (10). The other is obtained from Eq. (13) and has its origins in the δ-function part of $\langle V|a_{\mathbf{p}}|V\theta_{\mathbf{k}}^{(\mathrm{out})}\rangle$. The combined contribution of these two terms gives $D_n^{(1)}(\omega_q,\omega_p)=G_\rho g_\rho u(q)u(p)[2\omega_q\omega_p]^{-3/2}\times(\omega_q+\omega_p)$. We now use this value of $D_n^{(1)}(\omega_q,\omega_p)$ together with the previously computed value of $D_v^{(2)}(\omega)$ to obtain

$$D_v^{(4)}(\omega_p)=G_\rho\frac{u(p)}{(4\pi^2)^2}g_\rho^4\left(\frac{\omega_p}{2}\right)^{1/2}\int\frac{k^2dk\kappa^2d\kappa}{\omega_k^2\omega_\kappa^2(\omega_k+\omega_\kappa-\omega_p-i\eta)(\omega_k-\omega_p-i\eta)(\omega_\kappa-\omega_p-i\eta)}. \tag{17}$$

The next step in the iteration, is the evaluation of $D_n^{(3)}(\omega_q,\omega_p)$; Eq. (7a) gives many contributions to this order; we will separate $D_n^{(3)}(\omega,\omega')$ into two parts $[D_n^{(3)}(\omega,\omega')]_a$ and $[D_n^{(3)}(\omega,\omega')]_b$ in the following fashion: All contributions to $D_n^{(3)}$ which originate from integrals involving the amplitudes $D_v^{(0)}$ or $D_v^{(2)}$ will be grouped into $[D_n^{(3)}]_a$. The contributions that have their origin in integrals involving $D_n(\omega,\omega')$ itself to first order in g_ρ will be grouped into $[D_n^{(3)}(\omega,\omega')]_b$. We note that the former group is obtained from the one-meson $(V-\theta)$ part of the absorptive amplitude while the latter is due to the two-meson $(N-\theta-\theta)$ part. Our earlier stated objective will, therefore, be to examine whether $[D_n^{(3)}(\omega,\omega')]_a$ and $[D_n^{(3)}(\omega,\omega')]_b$ are separately finite.

We note that iteration gives

$$\begin{aligned}[D_n^{(3)}(\omega_q,\omega_p)]_a=\frac{-G_\rho g_\rho^3u(q)u(p)}{4\pi^2(8\omega_q\omega_p)^{1/2}}\int k^2dku^2(k)\bigg\{&\left(\frac{1}{\omega_q}+\frac{1}{\omega_p}\right)\frac{1}{\omega_k^2(\omega_k-\omega_q-\omega_p-i\eta)}\\&+\left(\frac{1}{\omega_k-\omega_q-i\eta}+\frac{1}{\omega_k-\omega_p-i\eta}\right)\frac{1}{\omega_k^2(\omega_k-\omega_q-\omega_p-i\eta)}\\&+\left(\frac{\omega_p}{\omega_q}\right)\frac{1}{\omega_k^3(\omega_k-\omega_p-i\eta)}+\left(\frac{\omega_q}{\omega_p}\right)\frac{1}{\omega_k^3(\omega_k-\omega_q-i\eta)}\bigg\},\end{aligned}$$

$$\begin{aligned}[D_n^{(3)}(\omega_q,\omega_p)]_b=\frac{G_\rho g_\rho^3u(q)u(p)}{4\pi^2(8\omega_q\omega_p)^{1/2}}\int k^2dku^2(k)\bigg\{&\frac{1}{\omega_k^3(\omega_k-\omega_p-i\eta)}+\frac{1}{\omega_k^3(\omega_k-\omega_q-i\eta)}\\&+\frac{1}{\omega_p\omega_k^2(\omega_k-\omega_q-i\eta)}+\frac{1}{\omega_q\omega_k^2(\omega_k-\omega_p-i\eta)}\bigg\}.\end{aligned}$$

It is apparent that both of these terms contain logarithmic divergences, in the sense that, as the cutoff implicitly contained in the function $u(k)$ recedes to infinity (in the limit of $u(k)=1$), the integrals become logarithmically infinite. $D_n^{(3)}(\omega_q,\omega_p)$, the sum of the two, however, is given by

$$D_n^{(3)}(\omega_q,\omega_p)=\frac{-G_\rho g_\rho^3 u(q)u(p)(\omega_q+\omega_p)}{4\pi^2(8\omega_q^3\omega_p^3)^{1/2}}\int\frac{k^2dku^2(k)[\omega_k(\omega_q+\omega_p)-(\omega_q^2+\omega_p^2)]}{\omega_k^2(\omega_k-\omega_p-i\eta)(\omega_k-\omega_q-i\eta)(\omega_k-\omega_q-\omega_p-i\eta)}$$

and remains finite even in the limit $u(k)=1$. $D_n^{(3)}(\omega_q,\omega_p)$ moreover agrees with the same value for this quantity as computed in the dressed particle picture[1] as well as by ordinary renormalized perturbation methods.

V. CONCLUSIONS

We can conclude from the preceding calculation that cancellations of divergent integrals between one and two-meson parts of decay amplitudes do occur. Although such cancellations do not take place in the evaluation of the (subtracted) equation for $D_v(\omega)$ [Eq. (6)], they enter already in the third-order iteration of the unsubtracted equation for $D_n(\omega,\omega')$. It is, of course, important to know whether this cancellation is a general feature of this method or whether it is specific to this model, particularly because the equation for $D_n(\omega,\omega')$ has no actual inhomogeneous parts, but is "driven" as it were by the $D_v(\omega)$ amplitude.[5] However, this feature of the calculation seems to be more specifically due to the linearity of the weak-decay Hamiltonian in the boson operator than to the limited number of intermediate states in the Lee model; it, therefore, seems likely to us that this calculation reflects something deeper than merely an aspect of the Lee model itself. It is, of course, necessary to undertake further investigations before such a conclusion can be drawn with any confidence; however, it would seem proper even on the basis of this evidence to regard this type of approximate treatment of absorptive decay amplitudes with caution.

[5] The authors are indebted to Professor S. B. Treiman for this remark.

VII. APPENDIX

In the Appendix we address ourselves to a question, raised in Sec. III in connection with the derivation of Eq. (12) from Eq. (11); namely, why in writing a dispersion relation for $\beta(\omega_q;\omega_p)$ we choose to contract on both boson operators instead of writing, much more simply,

$$\beta(\omega_q;\omega_p)=2^{-1/2}[\sum_{\mathbf{k}}\langle N\theta_q^{(\mathrm{out})}|a_{\mathbf{p}}|V\theta_{\mathbf{k}}^{(\mathrm{out})}\rangle D_v(\omega_k) +\sum_{\mathbf{k},\mathbf{k}'}\langle N\theta_{\mathbf{q}}^{(\mathrm{out})}|a_{\mathbf{p}}|N\theta_{\mathbf{k}}\theta_{\mathbf{k}'}^{(\mathrm{out})}\rangle D_n(\omega_k,\omega_{k'})]. \quad \text{(A1)}$$

This is of some interest, since Eq. (A1) besides being much simpler than Eq. (12) is entirely independent of any dynamical assumptions, whereas Eq. (12) follows in part from the linearity of H_w in the boson operator, and to that extent depends on the specific form of the Hamiltonian H_w.

As noted in the body of the paper, Eq. (A1) leads to a trivial and useless identity. Substitution of Eqs. (14)–(15c) into (A1), and symmetrization in ω_q and ω_p leads to

$$\beta(\omega_q,\omega_p)=D_n(\omega_q,\omega_p)-\alpha(\omega_q,\omega_p).$$

Equation (7), thus, reduces to the trivial identity $D_n(\omega_q,\omega_p)=D_n(\omega_q,\omega_p)$.